EFFECT OF ROCK POROSITY ON FORMATION RESISTIVITY FACTOR

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ABSTRACT

Formation resistivity factor (Fr) is defined as the resistivity of rock fully saturated with brine (R_o) divided by the resistivity of brine (R_w). This factor is function of porosity, resistivity, tortuosity and cementation factor, which are consequently affected also by particle size and degree of compaction. During the present paper we try to develop a new correlation between the formation resistivity factor (Fr) and the rock porosity taking into consideration the effect of water salinity, and confined pressure. Using the best fit curve and the best correlation coefficients are obtained to determine a new formation resistivity factor correlation. Using some synthetic core samples many empirical correlations were obtained to determine the effect of the porosity on the formation resistivity factor.

1. Introduction

Sedimentary formations are capable of transmitting an electric current only by means of the interstitial and adsorbed water they contain. They would be non-conductive if they were entirely dry.

The interstitial or connate water containing dissolved salts constitutes an electrolyte capable of conducting current, as these salts dissociate into positively charged cations, such as Na⁺ and Ca²⁺, and negatively charged anions, such as Cl⁻ and SO₄²⁻. These ions move under the influence of an electrical field and carry an electrical current through the solution. The greater the salt concentration, the greater the conductivity of the connate water. The electrical resistivity (reciprocal of conductivity) of a fluid-saturated rock is its ability to impede the flow of electric current through that rock. Dry rocks exhibit infinite resistivity. The resistivity of reservoir rocks is a function of salinity of formation water, effective porosity, and quantity of hydrocarbons trapped in the pore space [1]. Relationships among these quantities indicate that the resistivity decreases with increasing porosity and increases with increasing petroleum content. Resistivity measurements are also dependent upon pore geometry, formation stress, and composition of rock, interstitial fluids, and temperature. Resistivity is, therefore, a valuable tool for evaluating the producibility of a formation. A rock that contains oil and/or gas will have a higher resistivity than the same rock completely saturated with formation water and the greater the connate water saturation, the lower the formation resistivity.

Archie defined the formation resistivity factor F_r as [2]
\[ F_r = \frac{R_o}{R_w} \quad (1.1) \]

Where \( R_o \) is the resistivity of a formation which is fully saturated with water, \( R_w \) is the resistivity of the water. \( R_o \) will be greater than \( R_w \) and \( F_r \) will always be greater than unity. Figure (1.1) shows the qualitative effect of brine resistivity (assuming all other factors, such as porosity, cementation, and amount of shale remain constant) on \( (F_r) \) for limestone and clean sand, and shaly (dirty) sand. The formation factor is essentially constant for clean sand and limestone. For dirty or shaly sand, \( (F_r) \) decreases as brine resistivity, \( R_w \), increases; and although \( R_o \) increases, it does not increase proportionately because the clay in the water acts as a conductor.

![Figure (1.1) General relationship between formation factor \( F_r \) and brine resistivity \( R_w \) factor (Courtesy of Core Laboratories).][2]

This effect is dependent upon the type, amount, and manner of distribution of the clay in the rock Equation (1.1) is an important relation in well-log interpretation for locating potential zones of hydrocarbons. Several methods for determining the reservoir water resistivity have been developed, including: chemical analysis of produced water sample, direct measurement in resistivity cell, water catalogs, spontaneous potential (SP) curve, resistivity-porosity logs, and various empirical methods.

2. Literature Review
The value of \( F_r \) is one of the most important parameters in water saturation calculations. The presence of \( F_r \) or equivalent parameters in all different formulas of water saturation calculation such as Archie, Indonesia, Popoun, etc… indicate the important role of this parameter in original oil in place estimation of a field.

2.1. Theoretical Formula for \( F_r \)
The formation resistivity factor,
\[ F_r = a \phi^m \quad (2.1) \]
has theoretical derivation in some of the early literature and textbooks on well log analysis and core analysis. Most all published derivations start with the fundamental definition of formation resistivity factor.

\[ F_r = \frac{R_o}{R_w} \quad (2.2) \]

Where \( R_o \) is the resistivity of the porous media 100% saturated with a conductive fluid and \( R_w \) is the resistivity of the conductive fluid.

Each derivation requires a simplified model of the porous media using geometric shapes of pores, pore throats, and bulk volume that are easily described in terms of length and cross-sectional area for the conduction of ions through the model.

A general derivation similar to Amyx et al \[3\] (1960) is shown here. The definition of resistivity (\( R \)) of many materials is

\[ R = \frac{rA}{L} \quad (2.3) \]

Where \( r \) = resistance of the material.

\( A \) = the cross-sectional area perpendicular to ionic flow.

\( L \) = Length of the ionic flow path.

Using a cube of salt water, the resistance of the cube could be defined as

\[ r_w = \frac{(R_w L)}{A} \quad (2.4) \]

Where \( L \) and \( A \) describe the dimensions of the cube of water. A cube of porous media of the same dimension of the cube of water would have a lesser volume available for water. The matrix is assumed to be an insulator as such the portion of the cube. That can conduct ionic flow is only the pore space.

Therefore, an apparent cross-sectional area (\( A_a \)) and apparent flow path (\( L_a \)) are used. The resistance of the cube is

\[ r_2 = \frac{R_w L_a}{A_a} \quad (2.5) \]

By definition the resistivity of the cube of core saturated with water is

\[ R_o = \frac{r_2 A}{L} \quad (2.6) \]

Substituting the last two equations yields:

\[ R_o = \frac{R_w L_a A}{A_a L} \quad (2.7) \]

Using this definition of \( R_o \) in the \( F_r \) equation result in:

\[ F_r = \frac{L_a / L}{A / A_a} \quad (2.8) \]

which is the ratio of the apparent flow path to the length of the cube compared to the ratio of the apparent cross-sectional area to the cross-sectional area of the cube. The ratio of the lengths is proportional to tortuosity and is given the symbol \( a \), the tortuosity factor. The apparent cross-sectional area is assumed to be equal to the product of the actual area and the porosity of the porous media (\( \Omega A \)). Using this definition yields

\[ F_r = a / \Omega \quad (2.9) \]

Porosity has no power as such \( m \) can be seen as one. Several attempts have been made to obtain a universal formula relating porosity, formation resistivity, and cementation factor. If an electric current is passed through a block of non-conducting porous rocks saturated with a conducting fluid, only a portion of the pore space participates in the flow of electric current. Consequently, total porosity \( \Omega \) can be divided into two components such that \[4]:

\[ \Omega = \Omega_{ch} + \Omega_{tr} \quad (2.10) \]

Where \( \Omega_{ch} \) and \( \Omega_{tr} \) are, the flowing porosity associated with the channels and the porosity associated with the regions of stagnation (traps) in a porous rock, respectively. It seems that, \( \Omega_{ch} \) is corresponding to the 'effective porosity' used by Chilingarian and \( \Omega_{tr} \) is corresponding to the irreducible fluid saturation \[5].
(2.10) and (2.11) show that the electrical current can flow only through the channel indicated by C, while no current can flow through the trap indicated by T. In figure 2.1 the traps are of the dead-end type. The trap in figure 2.2 is called an open or symmetry trap. A universal relationship between $F_r$ and $\Phi_f$ may be written as $^6$:

$$F_r = 1 + f_G \left( \frac{1}{\Phi_{ch}} - 1 \right)$$

(2.11)

Figure 2.1 Portion of porous rock showing showing dead-end traps $^4$

Figure 2.2 Portion of porous rock an open or symmetry trap $^4$

$$\Phi_{ch} = \Phi^m$$

(2.12)

Where $f_G$ is defined as the interior geometry parameter of the porous rock, and $m \geq 1$. Combining Equations 2.11 and 2.12 gives:

$$F_r = 1 + f_G \left[ \frac{1}{\Phi^m} - 1 \right]$$

(2.13)

This is the Rosales relationship between formation resistivity, porosity, and cementation factor. If $f_G = 1$, Equation 2.13 gives Archie's formula. Equation 2.13 can be expressed as:

$$F_r = \frac{f_G}{\Phi^m} + (1 - f_G)$$

(2.14)

The value of $f_G$ for most porous rocks is close to unity. Hence, $f_G/\Phi^m >> (1-f_G)$ and Equation 2.14 can be approximated by:

$$F_r = \frac{f_G}{\Phi^m}$$

(2.15)

This expression is the Humble formula where $f_G = a$. Thus, Archie's formula and Humble's formula are special cases of Rosales general formula. Rosales showed experimentally that, for sandstones, Equation 2.15 can be written as follows $^6$:

$$F_r = 1 + 1.03 \left[ \frac{1}{\Phi^{1.73}} - 1 \right]$$

(2.16)

This expression was compared graphically with the Humble formula, Equation 2.17, and Timur et al. formula $^7$. 
Figure 2.3 is a log-log plot of Humble Equation (line A), 2.11 (line B), and 2.18 (line C). The three formulas give approximate results within the region of practical interest, i.e., 10 ≤ Ø ≤ 40. As Ø approaches unity, however, Equation (2.11) gives a curved line that satisfies the condition \( F_r = 1 \) when \( \Phi = 1 \), whereas Humble’s formula and Timur’s et al. formula (Equation 2.18) are straight lines for all values of \( \Phi \), which does not satisfy that condition.

\[
R_{wc} = \frac{E \frac{\Phi A(L/L_a)}{I_{wc}}}{L}
\]

(2.17)

\[
F_r = \frac{1.13}{\Phi^{1.73}}
\]

(2.18)

The tortuosity is:
\[
\tau = \Phi F_r
\]

(2.19)

Substituting Equation 2.13 into Equation 2.19 yields a general expression for calculating tortuosity:
\[
\tau = \Phi \left[ 1 + f_G \left( \frac{1}{\Phi^m} - 1 \right) \right]
\]

(2.20)

Inasmuch as the value of \( f_G \) is approximately equal to unity for most porous rocks, Equation 2.20 can be written as follows:
\[
\tau = \frac{1}{\Phi^{m+1}}
\]

(2.21)

Combining Equations 2.10, 2.12, and 2.21 gives:
\[
\tau = 1 + \frac{\Phi^m}{\Phi^m_{ch}}
\]

(2.22)

This expression indicates the physical significance of tortuosity in terms of stagnant and flowing porosities. Equation 2.22 is approximation valid only for consolidated
porous rocks. For unconsolidated sands, the general expression (Equation 2.20) should be used, where \( f_G = 1.49 \) and \( m = 1.09 \).

### 2.2 Formation Resistivity Factor, Fr and Porosity:
As clean sedimentary rocks conduct electricity by virtue of the salinity of water contained in their pores, it is natural that the porosity is an important factor in controlling the flow of electric current.

As a first approximation, one would expect that the current conductance would be no more than that represented by the fractional porosity, e.g., a formation with 20% connate water saturation and 80% oil saturation would be expected to transmit no more than 20% of the current that would be transmitted if the entire bulk volume conducted to the same degree as the water.\(^8\)

\[
F_r = \frac{1}{\phi}
\]

(2.23)

### 4. For Synthetic Cores Samples
Laboratory measurement results of an experimental study (Attia 2005)\(^9\) were used to make this research. The experimental measurements were made using 12 manufactured core samples. These samples were divided into four groups depending on the grain size; each group has three cores of the same grain size (the same degree of sorting) but was manufactured at different compaction pressures. Tables (1.1) through (1.4) show these four groups.

The first group table (1.1) consists of three cores (2, 6 and 14) have the same degree of sorting equals 100% manufactured at compaction pressures of 3000psi, 4000psi, 5000psi respectively. The porosity of the cores is 30%, 29% and 29% respectively. Similarly table (1.2) show the specifications of the second group which consists of 3 cores (18, 22 and 28) with degree of sorting of 80% and confined pressures of 3000psi, 4000psi and 5000psi and porosities of 28%, 27% and 25%. In the same manner table (1.3) and table (1.4) for the third and the fourth groups which contain core samples (32, 38 and 42) and (48, 54 and 58) respectively. These data were used to study the effect of the degree of sorting, porosity, and compaction pressure on the formation resistivity factor. The measurements of the formation resistivity factor were made at three different values of salinity 0.3%NaCl, 1%NaCl, and 5%NaCl to study also the effect of the formation water salinity on resistivity factor.

**Table (1.1): sandstone Fr at 5%NaCl, 0.3%NaCl, 1%NaCl. (Attia 2005)\(^9\)**

**Group 1:**

<table>
<thead>
<tr>
<th>Core #</th>
<th>Degree of Sorting</th>
<th>Porosity (Fraction)</th>
<th>Fr at 5% NaCl</th>
<th>Fr at 0.3% NaCl</th>
<th>Fr at 1% NaCl</th>
<th>Compaction Pressure (psi.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>100</td>
<td>0.3</td>
<td>5.64</td>
<td>3.15</td>
<td>5.80</td>
<td>3000</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>0.29</td>
<td>7.11</td>
<td>3.62</td>
<td>5.96</td>
<td>4000</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>0.29</td>
<td>7.52</td>
<td>3.91</td>
<td>6.09</td>
<td>5000</td>
</tr>
</tbody>
</table>
Table (1.2): sandstone Fr at 5%NaCl, 0.3%NaCl, 1%NaCl. (Attia 2005) [9]

Group 2:

<table>
<thead>
<tr>
<th>Core #</th>
<th>Degree of sorting</th>
<th>Porosity (Fraction)</th>
<th>Fr at 5% NaCl</th>
<th>Fr at 0.3% NaCl</th>
<th>Fr at 1% NaCl</th>
<th>Compaction Pressure (psi.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>80</td>
<td>0.28</td>
<td>6.78</td>
<td>3.57</td>
<td>6.85</td>
<td>3000</td>
</tr>
<tr>
<td>22</td>
<td>80</td>
<td>0.27</td>
<td>9.79</td>
<td>3.66</td>
<td>7.58</td>
<td>4000</td>
</tr>
<tr>
<td>28</td>
<td>80</td>
<td>0.25</td>
<td>10.2</td>
<td>4.08</td>
<td>7.75</td>
<td>5000</td>
</tr>
</tbody>
</table>

Table (1.3): sandstone Fr at 5%NaCl, 0.3%NaCl, 1%NaCl. (Attia 2005) [9]

Group 3:

<table>
<thead>
<tr>
<th>Core #</th>
<th>Degree of sorting</th>
<th>Porosity (Fraction)</th>
<th>Fr at 5% NaCl</th>
<th>Fr at 0.3% NaCl</th>
<th>Fr at 1% NaCl</th>
<th>Compaction Pressure (psi.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
<td>60</td>
<td>0.24</td>
<td>11.1/3</td>
<td>3.87</td>
<td>9.38</td>
<td>3000</td>
</tr>
<tr>
<td>38</td>
<td>60</td>
<td>0.24</td>
<td>13.7/5</td>
<td>4.12</td>
<td>9.76</td>
<td>4000</td>
</tr>
<tr>
<td>42</td>
<td>60</td>
<td>0.22</td>
<td>15.1/8</td>
<td>4.57</td>
<td>10.13</td>
<td>5000</td>
</tr>
</tbody>
</table>

Table (1.4): sandstone Fr at 5%NaCl, 0.3%NaCl, and 1%NaCl (Attia 2005) [9]

Group 4:

<table>
<thead>
<tr>
<th>Core #</th>
<th>Degree of sorting</th>
<th>Porosity (Fraction)</th>
<th>Fr at 5% NaCl</th>
<th>Fr at 0.3% NaCl</th>
<th>Fr at 1% NaCl</th>
<th>Compaction Pressure (psi.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>40</td>
<td>0.21</td>
<td>14.0/8</td>
<td>6.42</td>
<td>13.0/6</td>
<td>3000</td>
</tr>
<tr>
<td>42</td>
<td>40</td>
<td>0.20</td>
<td>21.1/4</td>
<td>6.51</td>
<td>13.1/9</td>
<td>4000</td>
</tr>
<tr>
<td>48</td>
<td>40</td>
<td>0.19</td>
<td>23.1/0</td>
<td>7.56</td>
<td>13.4/7</td>
<td>5000</td>
</tr>
</tbody>
</table>

5. Results and discussion

5.1. Effect of rock porosity on formation resistivity factor:

To predict a correlation between the formation resistivity factors as function of rock porosity similar to correlations that exist in literature, data of tables (1.1) through table (1.4) were plotted as shown in fig. (1.1) through fig. (1.3).

These figures show the correlation between the formation resistivity factor (Fr) and the rock porosity taking into consideration the effect of water salinity, and confined pressure.
From these figures it is noted that the formation resistivity factor increases with decreasing the rock porosity for different values of confined pressure and water salinity. These data were plotted also on a log-log scale as shown in fig.(1.4) through (1.6) to find the empirical correlations in the following general formula $F_r = a\Phi^b$. 

- 8 -
Figure (1.4) Effect of porosity on the formation resistivity factor at water salinity of 5% NaCl with different compaction.

Fr @5000 psi = 0.2069x^{2.9372}
R² = 0.9991

Fr @3000 psi = 0.2537x^{-2.5248}
Fr @4000 psi = 0.229x^{-2.9144}
R² = 0.97
R² = 0.9981

Figure (1.5) Effect of porosity on the formation resistivity factor at water salinity of 1% NaCl with different compaction.

Fr @ all = 0.4959x^{2.0984}
R² = 0.9962

Figure (1.6) Effect of porosity on the formation resistivity factor at water salinity of 0.3% NaCl with different compaction.

Fr @5000 psi = 0.3365x^{-1.778}
R² = 0.8897

Fr @ 3000 psi = 0.5065x^{-1.5297}
Fr @4000 psi = 0.4476x^{-1.6959}
R² = 0.8568
R² = 0.8424
These obtained correlations are given as shown in table (1.5) for the different values of formation water salinities and confined pressures.

**Table (1.5) effect of rock porosity on Formation resistivity factor.**

<table>
<thead>
<tr>
<th>Brine, salinity</th>
<th>Confined pressures (C)</th>
<th>Correlations</th>
</tr>
</thead>
</table>
| 5% NaCl         | 3000 psi. 4000 psi. 5000 psi. | \( F_r = 0.2537\phi^{-2.5249} \) (5.33)  
                       \( F_r = 0.229\phi^{-2.8144} \) (5.334)  
                       \( F_r = 0.2069\phi^{-2.9372} \) (5.335) |
| 1% NaCl         | 3000 psi. 4000 psi. 5000 psi. | \( F_r = 0.4959\phi^{-2.0604} \) (5.36) |
| 0.3% NaCl       | 3000 psi. 4000 psi. 5000 psi. | \( F_r = 0.5065\phi^{-1.5297} \) (5.37)  
                       \( F_r = 0.4476\phi^{-1.6959} \) (5.38)  
                       \( F_r = 0.3365\phi^{-1.778} \) (5.39) |

These correlations are more representative to the formation resistivity factor than those are presented in the previous literature studies because it take into consideration the effect of confined pressure, which is easy to obtain as a function of depth instead of cementation factor (m). Also these correlations include the effect of formation water salinity which was neglected in the previous formula. To simplify the use of these correlations we used the constant of the correlations obtained in table (1.5) and plotted the following two figures (1.7) and (1.8) to determine the values of the constant (a) and (b) of the general formula \( F_r = a\phi^{-b} \).

Constants (a) and (b) are functions of confined pressures and water salinities. Using the values of (a) and (b) for confined pressure and salinities then substitute in the general equation \( F_r = a\phi^{-b} \) we can find the formation resistivity factor as a function of porosity.

**Figure (1.7) Effect of Porosity**

![Figure 1.7 Effect of Porosity](image-url)
Conclusion
Based on the results obtained from this study, we can conclude the following:
1. The formation resistivity factor which is the ratio of the electrical resistivity of porous medium fully saturated with water to the water resistivity is very important factor in electric log interpretation.
2. Using the specifications of some synthetic cores manifested at different confined pressures with different porosities degree of sorting and water salinities, the effect of different factors on the formation resistivity were studied
3. The porosity increases, the formation resistivity factor decreases. The predicted empirical correlation for the formation resistivity factor was obtained as: Fr = a Ø^-b
   where a and b are constant depend on the confined pressure and the water salinity.
4. The constants a and b for the all predicted formula were determined and given as a graphs to be easily obtained with respect to its dependent factors.

References