Stability of Parallel DC/DC Converters with Time Varying Delay

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Abstract—The rapid development in communication networks made them very attractive to replace conventional wiring in distributed power electronic systems. Parallel DC/DC converters are one of the important distributed power electronic systems. Conventionally, the controllers exchange the control signals through wires. This reduces the reliability and increases the system cost and complexity. Replacing them with communication network will introduce a time delay that may lead to system instability. The paper addresses the stability of parallel DC converters controlled over communication networks. The network is represented as variable time delay. A theorem based on solving a set of linear matrix inequalities is used to investigate the delay-dependent stability of the system. The effects of the voltage controller parameters on the maximum allowable delay bound are investigated. Furthermore the effect of the time delay variation rate on the maximum allowable delay bound is discussed.

Index Terms—delay-dependent; DC converters; networked control system; parallel; stability; time delay

INTRODUCTION

Parallel DC/DC converters are found in many very important industrial applications such as DC Microgrids, UPS (Uninterruptable Power Supplies), interactive power electronic networks (IPNs), more electric aircrafts (MEAs), electric traction light rail transportation system and Electric Hybrid Vehicles. This parallel configuration for DC/DC converters offers many advantages such as increased power processing capabilities, improved reliability, easy maintenance, future expansion, and modularity [1,2]. The recent advances in low cost and high bandwidth communication networks made them attractive to replace the conventional wiring in the parallel DC/DC converters control. Replacing the wiring with communication network provides many advantages such as modularity, simplified wiring, low cost, reduced weight, decentralization of control, integrated diagnosis, simple installation, quick and easy for maintenance, flexible expandability and reconfigurability [3]. The proposed networks for control signal exchange are Control Area Network (CAN), Ethernet and wireless networks. These networks are characterized by time varying delay which can lead to system instability if they are not considered during the design stage.

The controller in parallel converters must achieve load sharing and voltage regulation. Different control strategies are used to achieve these tasks which can be classified into centralized, decentralized, quasi-decentralized and distributed control strategies. The main disadvantage of the centralized controller is the complexity and the single point of failure. The distributed control solves the problem associated with the complex centralized controller, but on the other hand the feature of good load sharing is lost. Quasi-decentralized control such as master-slave is the most preferable control strategy for the communication-based control method because it could achieve better load sharing and involves less use of the communication network compared with the centralized control strategy. Additionally, if a rotating master-slave control strategy is implemented the reliability of the system could be enhanced because in the case of the master failure one of the slaves could take his role.

Most of the published research work focus on estimating the maximum allowable delay bound (MADB) with constant time delay. The assumption of constant time delay cannot be guaranteed in communication networks where the time delay is time varying or even random. The published research is based on the frequency domain methods that can only be applied to constant time delay cases. In [4,5] the master-slave control strategy is implemented where the master current signal is distributed through an RF communication link. The MADB is estimated through frequency domain method. The method is based on replacing $s$ with $j\omega$ and sweeping the frequency from 0 to $\infty$ in order to find where the roots cross the imaginary axes. Although the method could provide less conservative results for the MADB, it can only be applied to the constant time delay case. In [6] the delay term is represented by first order Taylor series. The method can be applied only for small constant time delays. Increasing the time delay increase the approximation error and hence more...
conservative result for the MADB for systems with larger MADB. The frequency domain method is also used in [7] to find the maximum time delay margin for wireless network. The stochastic stability of the parallel DC/DC converters with Markovian random time delay is presented in [8].

The paper presents stability analysis of parallel DC/DC converters with constant and time-varying delay. The time delay and its variation are assumed to be bounded. The stability criteria are based on solving a set of linear matrix inequalities (LMIs) [9]. The method is based on using Newton-Leibnitz formula to replace the delay term, then Lyapunov-Krasovskii functional is used to derive a set of LMIs. The paper starts with modeling the parallel DC/DC Buck converters with varying time delay. The master-slave control strategy is adopted. The master controller has a PI voltage and a PI current controller where the output of the voltage controller is sent through a communication network to all the slaves in the network. The slave control contains a PI current controller and has to track the reference current with the presence of the varying time delay. The parallel DC/DC Buck converters and the controllers are then modeled as a nominal time delay system. The stability criteria of the system are then presented. A system of two parallel DC/DC Buck converters is used as a case study. The stability of the case study system with constant and time varying delay is finally discussed.

**PARALLEL DC/DC BUCK CONVERTERS WITH TIME DELAY**

A system consists of $n$ parallel connected DC/DC Buck converters is shown in Fig. 1. The converters are designed to supply power with a specified current and voltage to the load. The converters must be identical in order to have load sharing a small difference between them can result in a large current sharing error between the converters [10] hence a current sharing controller is necessary. The state-space time average model of a system consists of $n$ parallel DC/DC converters is given by:

$$\begin{align*}
\frac{d i_1}{dt} &= \frac{-r_{11}}{L_1} i_1 - \frac{1}{L_1} v + \frac{V_n}{L_1} d_1 \\
\frac{d i_2}{dt} &= \frac{-r_{22}}{L_2} i_2 - \frac{1}{L_2} v + \frac{V_n}{L_2} d_2 \\
&\vdots \\
\frac{d i_n}{dt} &= \frac{-r_{nn}}{L_n} i_n - \frac{1}{L_n} v + \frac{V_n}{L_n} d_n \\
\frac{d v}{dt} &= (1/\sum_{i=1}^{n} C_i)[i_1 + i_2 + \cdots + i_n] - \frac{1}{R} v
\end{align*}$$

where $i_1, \ldots, i_n$ are the inductor currents and $v$ is the output voltage which are chosen as the state variables for the system. $d_1, \ldots, d_n$ are the duty cycles of the converters which are chosen as the control inputs. The other parameters are shown in Fig 1. Writing (1) in matrix form:

$$\dot{x}(t) = A x(t) + B d(t)$$

where:

$$A = \begin{bmatrix}
-\frac{r_{11}}{L_1} & 0 & 0 & \cdots & -\frac{1}{L_n} \\
0 & -\frac{r_{22}}{L_2} & 0 & \cdots & -\frac{1}{L_n} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & \cdots & -\frac{r_{nn}}{L_n} & -\frac{1}{L_n} \\
1/\sum_{i=1}^{n} C_i & 1/\sum_{i=1}^{n} C_i & \cdots & 1/\sum_{i=1}^{n} C_i & -1/\sum_{i=1}^{n} R C_i
\end{bmatrix}$$

$$B = \begin{bmatrix}
\frac{V_n}{L_1 V_n} & 0 & \cdots & 0 \\
0 & \frac{V_n}{L_2 V_n} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \frac{V_n}{L_n V_n} \\
0 & 0 & \cdots & 0
\end{bmatrix}$$

Figure 1. A parallel DC/DC converters

The master-slave control strategy for two converters is shown in Fig. 2. The reference current signal is sent through a communication network. This signal will suffer time delay and data loss which degrades the system performance or at worst leads to system instability.

![Communication-based master-slave control strategy](image)

The master and slave modules are stabilized by PI controllers.
The time-varying delay is $\tau(t)$. The controllers represented by (3) and (4) can be written as:

$$d_z = V_{x_{\tau}\rightarrow y} - v$$

$$\frac{dz_1}{dt} = K_{\mu}z_1 + K_{\rho}[V_{x_{\tau}\rightarrow y} - v] - i_{\tau}$$

$$\frac{dz_2}{dt} = K_{\rho}z_2(t - \tau(t)) + K_{\rho}[V_{x_{\tau}\rightarrow y} - v(t - \tau(t))] - i_{\tau}$$

$$\frac{dz_3}{dt} = K_{\rho}z_3(t - \tau(t)) + K_{\rho}[V_{x_{\tau}\rightarrow y} - v(t - \tau(t))] - i_{\tau}$$

$$\frac{dz_4}{dt} = K_{\rho}z_4(t - \tau(t)) + K_{\rho}[V_{x_{\tau}\rightarrow y} - v(t - \tau(t))] - i_{\tau}$$

The duty ratio of the converters:

$$d_k = \frac{K_{\rho}z_k(t - \tau(t)) + K_{\rho}[V_{x_{\tau}\rightarrow y} - v(t - \tau(t))] - i_{\tau}}{K_{\rho}}$$

The controllers represented by (3) and (4) can be written as:

$$\dot{z}(t) = E_{z_1}(t) + F_{z_2}(t) + E_{z_2}x_2(t - \tau(t)) + F_{z_3}z_3(t - \tau(t))$$

$$u_2(t) = C_{x_2}(t) + D_{x_2}(t) + C_{x_2}x_2(t - \tau(t)) + D_{x_2}z_3(t - \tau(t))$$

Applying the controller (6) into the plant (2), we get:

$$\dot{x}(t) = A_{x}(t) + A_{x}(t - \tau(t))$$

$$+ B_x[C_{x_2}(t) + D_{x_2}(t) + C_{x_2}x_2(t - \tau(t)) + D_{x_2}z_3(t - \tau(t))]$$

$$z(t) = E_{x_2}(t) + F_{z_3}(t) + E_{z_3}x_3(t - \tau(t)) + F_{z_4}z_4(t - \tau(t))$$

Writing (7) and (8) in matrix form we get:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \end{bmatrix} = \begin{bmatrix} A_{x} + B_xC & B_xD \\ E & F \end{bmatrix} \begin{bmatrix} x_2(t) \\ z(t) \end{bmatrix}$$

Equation (9) can be further written as:

$$\begin{bmatrix} \dot{x}(t) \\ z(t) \end{bmatrix} = A_{x} + A_{x}(t - \tau(t))$$

where;

$$A = \begin{bmatrix} A_x + B_xC & B_xD \\ E & F \end{bmatrix}$$

## Delay-Dependent Stability of the Parallel DC/DC Buck Converters

Control systems can be classified into delay-independent control systems and delay-dependent control systems. In the delay-independent systems the system is stable regardless of the time delay. Contrary, in the delay-dependent systems, there is a margin for the time delay. Most of the real-time control systems are delay-dependent control systems. The methods for analyzing the stability of time delay systems can be divided into the frequency domain and time domain methods. In the frequency domain methods the time delay is assumed to be constant which is not the case in reality. The MADB is not sufficient to guarantee the stability of the time delay system if the time delay is time varying. The time delay variation can lead itself to system instability. The time domain delay stability methods can deal with both constant and time varying delays. These methods are based on using Lyapunov-Krasovskii functional and Razumikhin functional. The method used in this paper is based on Lyapunov-Krasovskii functional and replacing the delay term by Newton-Leibnitz formula. Then the stability problem is formulated as a set of LMI s. The parallel DC/DC Buck converters with time delay can be represented as:

$$\dot{x}(t) = A_{x}(t) + A_{x}(t - \tau(t))$$

$$x(t) = \Phi(t)$$

$$t \geq 0$$

$$t \in [-\rho,0]$$

Where $A$ and $A_{x}$ are constant matrices with appropriate dimensions. $\Phi(t)$ is the initial condition of the system for the time interval $t \in [-\rho,0]$. The time-varying delay is $\tau(t)$ and satisfies the following:

$$0 \leq \tau(t) \leq \rho, \dot{\tau}(t) \leq \mu \leq 1$$

Where $\rho$ is the upper bound of the time delay, $\mu$ is the upper bound of the varying rate of the time delay.
Theorem 1 [9]: Given scalars $\rho > 0$ and $\mu > 0$, the time-delay system (11) is asymptotically stable if there exist symmetric positive-definite matrices $P = P^T > 0$, $Q = Q^T > 0$ and $Z = Z^T > 0$, a symmetric semi-positive-definite matrix $X = \begin{bmatrix} X_{11} & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix} \geq 0$, and any appropriate dimensioned matrices $Y$ and $T$ such that the following LMIs are true:

$$
\Phi = \begin{bmatrix}
\Phi_{11} & \Phi_{12} \\
\Phi_{12}^T & \Phi_{22}
\end{bmatrix} < 0
$$

where $\Phi_{11} = \rho A^T Z$, $\Phi_{12} = \rho A^T Y$, $\Phi_{22} = \rho A^T Z - \rho Z$,

$$
\Psi = \begin{bmatrix}
X_{11} & X_{12} \\
X_{12}^T & X_{22} \\
Y^T & T^T
\end{bmatrix} \geq 0
$$

and the following algorithm is used to solve the LMIs in theorem 1.

**Step 1:** Using the parameters set, obtain the system matrices: $A_0$, $B_0$, $E$, $F$, $E_d$, $F_d$, $C$, $D_c$ and $D_d$. Then calculate $A$ and $A_d$. The dimension of $A$ and $A_d$ depends on the number of the converters, in this case they are 6-by-6. In accordance the variable matrices $P$, $Q$, $Z$, $Y$, $T$, $X_{11}$, $X_{12}$, and $X_{22}$ are 6-by-6.

**Step 2:** Solve the LMIs in theorem 1 using the LMI toolbox in Matlab, for a constant time delay case $\mu = 0$. After defining the LMIs the `feasp` function is used to test whether the LMIs are feasible with the given time delay or not.

**Step 3:** To find MADB there are many methods that can be used. The simplest one is to start with setting $\tau = 0$ and increment the delay with a step $\Delta \tau$. Using smaller values of $\Delta \tau$ results in more accurate results, however the time for the solution takes longer. The binary algorithm can be used to give faster solution [1]. The idea is to set an interval $[\tau_n, \tau_{n+1}]$, where $\tau_n$ corresponds to unfeasible value and $\tau_{n+1}$ corresponds to a feasible value of time delay. The new value is $\frac{\tau_n + \tau_{n+1}}{2}$, then the interval is reduced depending on $\tau_{n+1}$ if $\tau_n$ is feasible, then $\tau_{n+1} = \tau_n$, and if it is unfeasible $\tau_n = \tau_{n+1}$, the iterations continue until we reach to the predetermined accuracy defined by $\epsilon = \xi - \tau_{n+1}$, where $\epsilon$ is error tolerance.

**Case Study**

A two parallel DC/DC converters are used to test the results of the stability method. The matrices describing the two DC/DC Buck converters and the two controllers are given as:

$$
A_0 = \begin{bmatrix}
-\frac{r_{01}}{L_0} & 0 & -\frac{1}{L_1} \\
0 & -\frac{r_{02}}{L_1} & -\frac{1}{L_1} \\
\frac{1}{R(C_1 + C_2)} & \frac{1}{R(C_1 + C_2)} & 0
\end{bmatrix}
$$

$$
B_0 = \begin{bmatrix}
V_{m0} \\
L_{V_m} \\
0
\end{bmatrix}
$$

$$
E = \begin{bmatrix}
0 & 0 & -K_{p0} \\
0 & -1 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

$$
F_0 = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
K_{p0} & 0 & 0
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
-K_{p} & -K_{p} & 0 \\
0 & -K_{p} & 0 \\
0 & 0 & -K_{p}
\end{bmatrix}
$$

$$
D = \begin{bmatrix}
K_{p} & K_{p} & K_{p} & 0 \\
0 & 0 & K_{p} \\
K_{p} & 0 & 0 & 0
\end{bmatrix}
$$

$$
C_e = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
$$

The parameters of the system are: $V_{m0}=10$ V, $C_1=C_2=220 \mu F$, $L_0=L_1=330$ $\mu H$, $R=3$ $\Omega$, $V_{m0}=1$ V, the switching frequency is 20 kHz, for comparison the controller parameters are chosen as [7]: $K_{p0}=0.28$, $K_{p0}=264$, $K_{p0}=0.106$, $K_{p0}=410$. For estimating the MADB with a given time delay variation rate, the output voltage, $V$, the switching frequency $f_s$, the input voltage $V_{in}$, the inductance $L_m$, and the time delay $\tau$, the following algorithm is used to solve the LMIs in theorem 1.

**Figure 3.** The output voltage with different time delays.
The current, A
Time (seconds)

Figure 4. (a) The master current controller, (b) The slave current controller

From Fig. 4. The current sharing is achieved with a steady-state current of 0.833 A. The oscillation in the current is resulting from the time delay. After 0.15 s the system reaches steady-state with negligible current sharing error, however the current sharing error exists in the transient. The models used in the simulation are the time varying nonlinear model in the SymPowerSystems toolbox in Matlab Simulink. The time delay margin obtained through simulation is 5.6 ms which is the same value obtained using frequency domain methods as reported in [7]. However, through simulating the linearized model the MADB obtained is 5.2 ms.

The MADB as a function of the controller parameters is investigated. The MADB as a function of the proportional and the integral gain of the PI voltage controller are shown in Fig. 5 using theorem 1. As can be seen the MADB depends strongly on $K_p$. As $K_p$ increases the MADB decreases. For small values of $K_p$ the MADB depends on $K_i$. For fixed $K_p$ we noticed that the MADB increases with $K_i$, then at a specific value of $K_i$ MADB becomes maximum then the MADB starts to decrease with increasing $K_i$. This shows that there is a specific value of $K_i$ that maximizes the MADB. This can be used to tune the PI controller, practically to achieve MADB with specific performance parameters. If $K_i$ is chosen to be large then $K_i$ has less effect on MADB, or in other words the MADB becomes less dependent on $K_i$. Additionally, for large values of $K_i$ the MADB changes slowly with $K_p$.

The MADB as a function of the time delay variation rate is shown in Fig. 6. It can be seen the MADB decreases as the time delay variation rate increase. Increasing the variation rate from 0 (constant time delay) to 1 will result in decreasing the MADB from 3.7 ms to 0.25 ms. From this we can conclude that not only the bound of the time delay that is important but also its variation rate. Fig. 6. Proves that the constant time delay criteria cannot guarantee the stability with time varying delay.

The effect of the time delay variation rate and the proportional gain of the PI voltage controller gain is also investigated and is shown in Fig. 7. The value of $K_{p0}$ is fixed at 264. We noticed that the MADB decreases with increasing the time delay variation rate. For fast varying time delay the MADB becomes less dependent on $K_{p0}$.

In order to test the results obtained from theorem 1 for a system with varying delay a simulation is carried out. From Fig. 6 for maximum time delay of 1.757 ms the maximum allowable time delay variation rate is 0.5 s/s. The time delay is then chosen to be: $\tau(t) = 0.8785(1 + \sin(285t))$ ms, with bounds $\tau_{MADB} = 1.757$ ms and $\mu = 0.5$ s/s. The output voltage is shown in Fig. 8, which shows that the stability is guaranteed even with the time-varying delay.

Figure 5. The MADB as a function of $K_{p0}$ and $K_{i0}$ using theorem

Figure 6. The MADB as a function of the time delay variation rate
strategy over communication network. The communication network is represented as varying time delay. The parallel DC/DC Buck converters are modelled as a standard time delay system. A theorem in the form of LMIs is used to estimate the MADB for the system. The delay-dependent stability method is based on using Lyapunov-Krasovskii functional and replacing the delay term by Newton-Leibnitz formula. The set of the LMIs is solved using Matlab with the binary search algorithm. The theorem is used to estimate the MADB and the results are tested through simulation. The effect of the PI voltage controller gains on the MADB is investigated and its found that the MADB depends strongly on Kp0. The effect of the time delay variation rate on the MADB is discussed and it is found increasing the time delay variation rate decreases the MADB. The stability of the system is tested with sinusoidal time varying delay.

V. CONCLUSION

The paper presents a delay-dependent stability of parallel DC/DC Buck converters implementing master-slave control strategy over communication network. The communication network is represented as varying time delay. The parallel DC/DC Buck converters are modelled as a standard time delay system. A theorem in the form of LMIs is used to estimate the MADB for the system. The delay-dependent stability method is based on using Lyapunov-Krasovskii functional and replacing the delay term by Newton-Leibnitz formula. The set of the LMIs is solved using Matlab with the binary search algorithm. The theorem is used to estimate the MADB and the results are tested through simulation. The effect of the PI voltage controller gains on the MADB is investigated and its found that the MADB depends strongly on Kp0. The effect of the time delay variation rate on the MADB is discussed and it is found increasing the time delay variation rate decreases the MADB. The stability of the system is tested with sinusoidal time varying delay.

REFERENCES


