Recursive Blind Adaptive Equalization for 16-QAM applications

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Abstract:
An innovative blind adaptive identification algorithm based on least-squares type arguments is offered in this project. Voided of any matrix inversion operation, parameter estimates are recursively updated with each output measurement. It is confirmed that the parameter estimates converge almost surely (a.s.) toward a scalar multiple of the true parameters. The viable utilization of this algorithm to the channel equalization problem is explained in this paper [3].

Introduction
Linear amplitude and phase dispersion in the channel primarily due to multipath resulting an ISI. Receivers must remove ISI to improve their systems performance [4]. Channel identification and equalization are vital to eliminate the results of ISI and attain high-speed reliable communication. Equalization can be accomplished either by designing the equalizer due to a priori knowledge of the channel. A second method of equalization commonly used is by sending training sequence. The method of assuming a priori for a channel in a radio communication environment is also not suitable, due to a small percentage of transmission time being used for a training sequence. A training sequence is no longer needed in the blind equalization methods. When access to the symbols being transmitted is excluded, the statistical properties of the transmitted signals are utilized to perform the equalization at the receiver. For example: the equalizer is chosen so that the statistics of the equalized output sequence \( \{ \hat{s}_k \} \) are close to the statistics of the source symbol sequence \( \{ s_k \} \). The scheme which is shown in Fig.1 is this example of Blind Equalization [2].

![Fig. 1 Schematic of blind equalization](image)

Problem Statement
Fractionally spaced samples of a single baseband received signal produce a single input multiple output model is a notable property in communication systems. For convenience of representation, we consider the following typical model of a single-input double-output system with two measurements \( y_1(i) \) and \( y_2(i) \), for each unknown transmitted symbol \( s(i) \)[2].

\[
y_1(i) = B(q^{-1}) s(i) + w_1(i) \\
y_2(i) = C(q^{-1}) s(i) + w_2(i) \\
\]

In which \( i = 0, 1, \ldots \) and \( q^{-1} \) is a unit delay operator. \( w_1(i) \) and \( w_2(i) \) are the measured noise sequences, respectively \( B(q^{-1}) \) and \( C(q^{-1}) \) denote two parallel channels given by:

\[
B(q^{-1}) = b_0 + b_1 q^{-1} + \ldots + b_q q^{-i} \\
C(q^{-1}) = c_0 + c_1 q^{-1} + \ldots + c_q q^{-i} \\
\]

Where \( L \geq 0 \) is the channel order. The objective is to identify the unknown parameters of \( B(q^{-1}) \) and \( C(q^{-1}) \) within only the noisy measurements \( y_1(i) \) and \( y_2(i) \). Without loss of generality, assume that \( s(i), w_1(i) \) and \( w_2(i) \) are zero for \( i < 0 \).

Recursive Parameter Estimation
Blind adaptive identification algorithm is offered in this section [2]. Let

\[
\varphi(i)^T = [y_1(i), y_1(i-1), \ldots, y_1(i-n_1), \nonumber \]

\[
y_2(i-1), \ldots, y_2(i-n_2)] \\
\]

And

\[
\theta^* = \theta B_0[c_0, c_1, \ldots, c_L, b_0, \ldots, b_L, 0, \ldots, 0] \\
\]

\( n_1 \) and \( n_2 \), which with the unknown degree \( L \) control the number of zeros in (3), are the known in this algorithm which recommend to produce \( \hat{\theta}(i), i \geq 0 \), an estimate of \( \theta \).

\[
\hat{\theta}(i) = \delta_{\theta}(i) R(i) / \theta(i-1) + R(i) / \theta(i) = ip(i) \\
\]

Where \( \delta_{\theta}(i) \) is an estimate of \( \sigma_{\varphi}^2 \).

\[
\left( i \right)^{\frac{1}{2}} = \frac{1}{\delta_{\theta}(i)} \left[ \varphi(i) \right]^{\frac{1}{2}} \\
\]

\[
p(i) = p(1 - i) - p(1 - i) \varphi(i) / \left( 1 + \varphi(i) / \varphi(i) - p(1 - i) \varphi(i) \right) \\
\]

With initial conditions \( \varphi(0) = \theta(0) \), and \( p(0) = p_0 \), \( p_0 > 0 \). By making some appreciations and we can neglect the difference between \( \hat{\theta}(i) \) and \( \hat{\theta}(i-1) \) when \( i \rightarrow \infty \). Will get \( p(i)^{-1} = p(0)^{-1} + \tilde{p}(i)^{-1} \). The impact of \( P(0) \) disappears as \( i \rightarrow \infty \), \( \varphi(i) \) and \( \tilde{p}(i) \) have similar asymptotic behavior which leads to motivation for the recursive algorithms [3].

Application to blind adaptive equalization
Based on the noisy measurements \( y_1(i) \) and \( y_2(i) \), where \( i \) is equal or greater than zero, the input sequence \( \{ s(i) \} \) must be recovered and for this objective we stack \( N \geq L \) into

\[
Y(i)^T = [y_1(i), y_2(i), \ldots, y_1(i-N), y_2(i-N+1)] \\
\]

\[
N: \text{The smoothing factor. Now from (1)} \\
Y(i) = HS(i) + W(i) \\
\]

(9)
Where
\[
S(i)^T = b_0 [s(i), s(i-1), \ldots, s(i-L-N+1)]
\]
\[
W(i)^T = [w_i(i), \ldots, w_i(i-L-N+1), w_{i+1}(i), \ldots, w_{i+L}(i-N+1)]
\]
\[H(i) = 2N \times (L+1)\]
\[
\begin{bmatrix}
 b_0 & b_1 & \cdots & b_L & 0 & 0 & \cdots & 0 & 0 \\
 0 & b_0 & \cdots & b_{L-1} & b_L & 0 & \cdots & 0 & 0 \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 0 & 0 & \cdots & \cdots & \cdots & \cdots & b_{L-1} & b_L \\
 c_0 & c_1 & \cdots & c_L & 0 & 0 & \cdots & 0 & 0 \\
 0 & c_0 & \cdots & c_{L-1} & c_L & 0 & \cdots & 0 & 0 \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 0 & 0 & \cdots & \cdots & \cdots & \cdots & c_{L-1} & c_L \\
\end{bmatrix}
\]
\]
\[H = \begin{bmatrix}
 b_0 & b_1 & \cdots & b_L & 0 & 0 & \cdots & 0 & 0 \\
 0 & b_0 & \cdots & b_{L-1} & b_L & 0 & \cdots & 0 & 0 \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 0 & 0 & \cdots & \cdots & \cdots & \cdots & b_{L-1} & b_L \\
 c_0 & c_1 & \cdots & c_L & 0 & 0 & \cdots & 0 & 0 \\
 0 & c_0 & \cdots & c_{L-1} & c_L & 0 & \cdots & 0 & 0 \\
 \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
 0 & 0 & \cdots & \cdots & \cdots & \cdots & c_{L-1} & c_L \\
\end{bmatrix}
\]

The optimal linear LMS estimator \( \hat{S}_0(i) \) of \( S(i) \) given \( Y(i) \) is given by [1]
\[
\hat{S}_0(i) = \sigma^2 S(i)^H (H H^* \sigma^2 + \sigma_0^2 I)^{-1} Y(i)
\]
\[(11)\]
Since matrix \( H \) is unknown, instead of (11) we use
\[
\hat{S}(i) = \sigma^2 \hat{H}(i)^* \hat{H}(i)^\dagger \sigma_z^2 + \sigma_0^2 I)^{-1} Y(i)
\]
\[(12)\]
Where \( \hat{H}(i) \) is obtained from the Sylvester matrix by substituting \( b_k \) and \( c_k \) is replaced in (12) with an estimate \( \delta_{\omega} \).

\[
\hat{S}(i) = \sigma^2 \hat{H}(i)^* \hat{H}(i)^\dagger \sigma_z^2 + \delta_{\omega}^2 I)^{-1} Y(i)
\]
\[(13)\]
When
\[
\sigma_\omega = \sigma_\omega \delta_{\omega}(i) \rightarrow H \quad (a.s.),
\]
and therefore
\[
\lim_{i \rightarrow \infty} \left( \frac{\hat{S}(i) - S_0(i)}{\delta_{\omega}} \right) = 0 \quad (a.s.).
\]
From (9), we can notice that \( P := E(Y(n)Y(n^*)) = HH^* \sigma_z^2 I + \sigma_0^2 I \), and if \( N > L \), then \( HH^* \) is a positive matrix with \( N - L \) eigenvalues equal to zero. This yield to the that \( N - L \) smallest eigenvalues of \( P \) are equal to \( \sigma_z^2 \). So by this we could find an estimate \( \delta_{\omega} \).

The simulation example and the results
The initial symbol is modulated using the common 16-quadrature amplitude modulation (16-QAM) constellation, and the channel impulse response is given by:
\[
h(t) = \left( 3 c(0.1t, t) - 1.5 e^{-\frac{t}{\tau}} c(0.1, t - \frac{T}{2}) \right) w_{4T}(t)
\]
Where
\[
c(\beta, t - t_0) \text{ is a raised cosine pulse with roll off factor } \beta \text{ and delay } t_0, \text{ while } w_{4T}(t) \text{ is rectangular window of duration } 4T, \text{ with } T \text{ being the symbol interval. By sampling twice the symbol rate, we obtain a single input two output systems of order three, i.e}
\]
\[
B(q^{-1})=0.6233+1.9054q^{-1}+(0.6064+0.75 i) q^{-2}-0.6233q^{-3}
\]
and
\[
C(q^{-1})=0.2699-0.1558+2.1749+0.4764i)q^{-1}+(-0.8251+0.4763i) q^{-2}+(0.2699-0.1558)q^{-3}
\]
where \( \tau^2 = -1 \), and both \( B(q^{-1}) \) and \( C(q^{-1}) \) are no minimum phase filters. Parameters are as follows: \( n_1 = 3 \) and \( n_2 = 4 \) in equation (11), while in (8)
\[N = 3. \text{ The noise variance is assumed to be well estimated, i.e. } \sigma_\omega = \sigma_\omega \). All the following figures are result of a single realization. In Fig. 2,3,4 denote \( \hat{\theta}(i) = \theta - \theta \), and the estimation error is shown in dB form by
\[
20 \log_{10} \| \hat{\theta} \|
\]
Fig. 2,3 show the norm of parameter estimation error for different values of noise variance. It is not difficult to see that the algorithm converges much faster with a smaller estimation error when \((a1)\) in the algorithm which is equal to \( \sigma_\omega^2 \) is smaller.

Fig. 4 and 5 present the magnitude of components of \( \hat{\theta}(i) \) using the algorithm for the same values of noise variance. According to (4), since \( n_2 = 4 \), the last element of \( \theta \) approaches 0, i.e. \( \lim_{i \rightarrow \infty} \hat{\theta}_0(i) = 0 \). From both Fig. 4 and 5, all the curves converge to their theoretical values, but the convergent rate of the second graph when \( \sigma_\omega^2 \) is smaller is faster.
Fig. 5. Magnitude of components of $\hat{\theta}(i)$ with a smaller $\hat{\sigma}_w^2$.

Fig. 6 and 7 show the eye diagram of $y_1(i)$, $i \geq 0$ (The output of the unequalized channel). Clearly, the inter-symbol interference is severe and a high error rate is expected.

Fig. 6. Eye diagram of $y(i)$

Fig. 7. Eye diagram of $y(i)$ with a smaller $\hat{\sigma}_w^2$.

The symbol was then transmitted and the equalized channel output is shown in Fig. 8, and 9, which indicates that the channel is well equalized. The rotation and magnification in the eye diagram is a function of $b_0 = -0.6322$.

Fig. 8. Eye diagram of the equalizer.

Fig. 9. Eye diagram of the equalizer with a smaller $\hat{\sigma}_w^2$.

To obtain a performance measure of the channel estimation, the mean-square error (MSE) of the estimator is used. From Fig. 10, and 11 present the Mean Square Error, it is clear that the MSE smaller with a smaller $\hat{\sigma}_w^2$.

Fig. 10. The Mean Square Error.

Fig. 11. The Mean Square Error with a smaller $\hat{\sigma}_w^2$.

Now, we repeat the experiment by replacing $b_0$ and $c_0$ in our single input two output systems.

$$B(q^{-1}) = 0.2699 - 0.1558i + 1.9054q^{-1} + (0.6064 + 0.75i)q^{-2} - 0.6233q^{-3}$$
$$C(q^{-1}) = -0.6233 + (2.1749 + 0.4764i)q^{-1} + (-0.8251 + 0.4763i)q^{-2} + (0.2699 - 0.1558i)q^{-3}$$
The recursive parameters estimation is developed for adaptive parameter identification when the reference signal is unknown. Instead of requiring independent identical distributed i.i.d. property of the input signal, we only assume that the input signal and noises are uncorrelated and the noises are a white noise sequences.

The algorithm is well designed, the almost sure convergence property of this algorithm for parameter estimates is presented. The proposed algorithm converges fast which can be seen from the simulation findings, which is very important in real time applications.

**References**


4- X. Li and H. Fan, “Direct Estimation of Blind Zero-Forcing Equalizers Based on Second-Order Statistics,” *IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 48, NO. 8, AUGUST 2000*